Taming turbulence via spectral nudging

Patricio Clark Di Leoni
University of Rome “Tor Vergata”

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In collaboration with:
Andrea Mazzino & Luca Biferale

UNIVERSITA' degli STUDI di ROMA
TOR VERGATA

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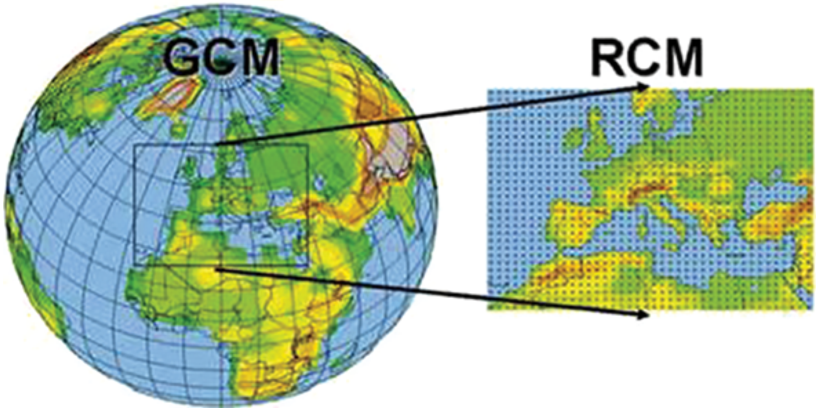
What is nudging?

- It is a *Data Assimilation* technique that can be used to control the evolution of a chaotic flow and improve its predictability [Lakshmivarahan & Lewis (2013)]
- It is also used to incorporate data coming from a Global Circulation Model into a Regional Climate Model [Waldron et al. (1996)]



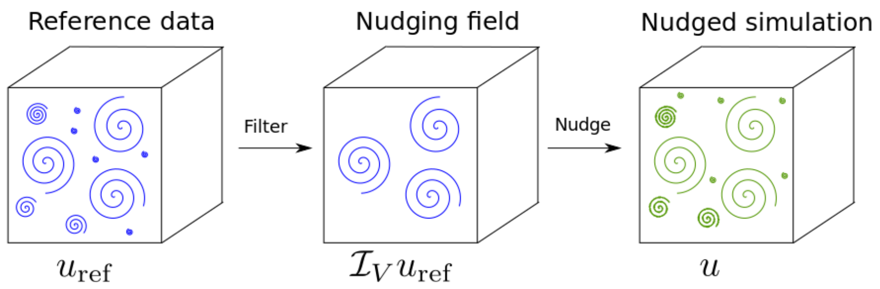
- Can we use nudging to probe the Eqs of motion?
- Is nudging strong enough to tame a 3D turbulent flow?
- What can we learn about turbulence in the process?

What is nudging? (with equations)

Nudged simulation

Navier-Stokes plus a penalty term

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\nabla p + \nu \nabla^2 u - \alpha \mathcal{I}_V (u - u_{\text{ref}})$$



Spatial filtering

Fourier low pass filter

$$u(x) = \sum_{-k_{\max}}^{k_{\max}} \hat{u}(k)^{-ik\cdot x} \qquad \mathcal{I}_V u(x) = \sum_{-k_c}^{k_c} u_k e^{-ik\cdot x}, \quad k_c < k_{\max}$$

Easy to implement in a pseudo-spectral code with periodic boundary conditions and $k_{\max} = N/3$

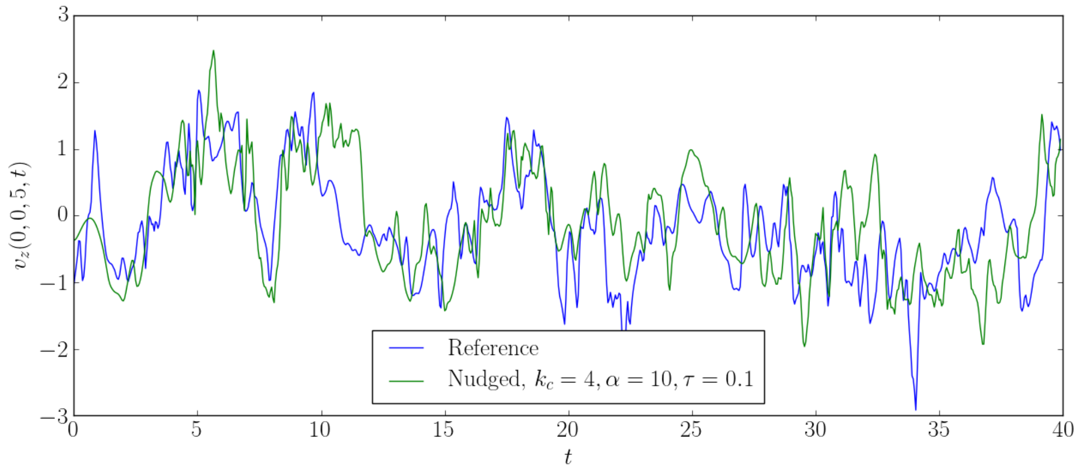
Reference data

We can generate our “true” data by performing simulation of the Navier-Stokes equations

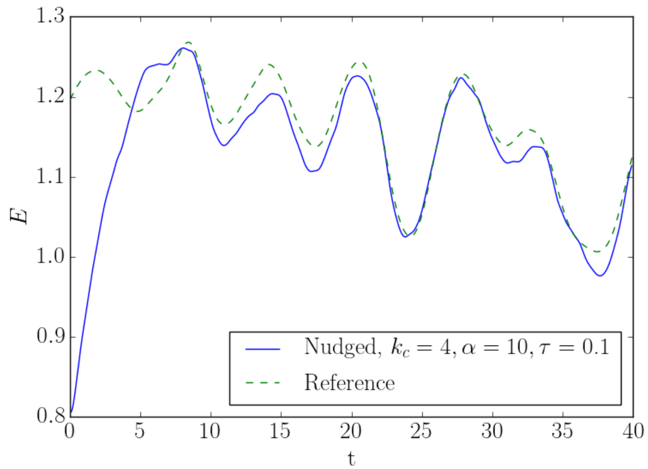
$$\frac{\partial u_{\text{ref}}}{\partial t} + (u_{\text{ref}} \cdot \nabla) u_{\text{ref}} = -\nabla p + \nu \nabla^2 u_{\text{ref}} + f$$

How nudging looks like

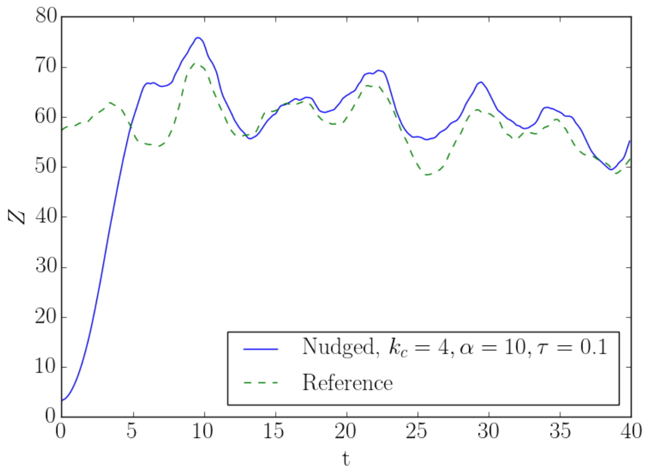
Velocity evolution



Energy



Enstrophy

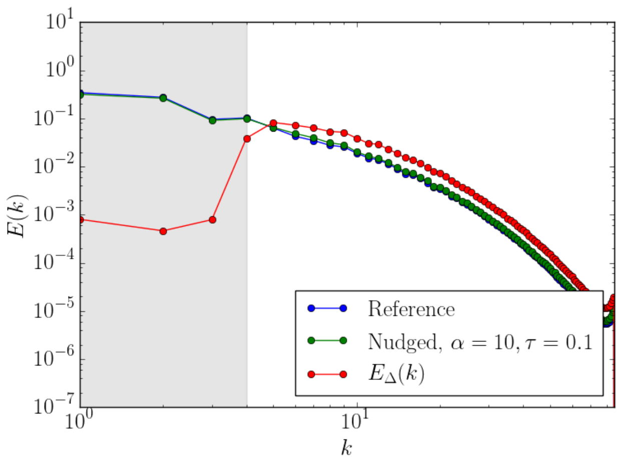


Easier to control the larger scales (energy) than the smaller ones (enstrophy)

Comparing the simulations

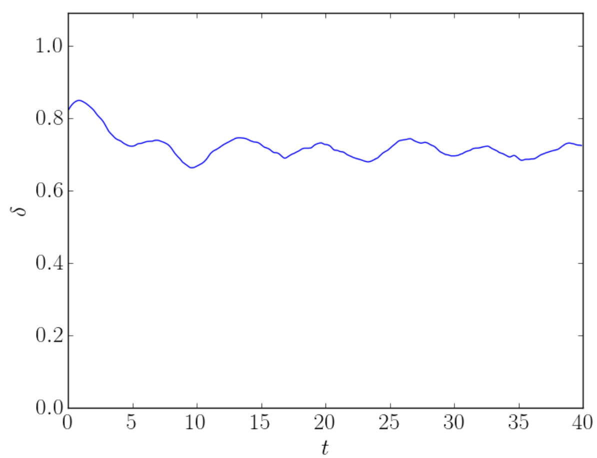
In our numerical set-up we can access the smaller scales of the reference flow. This way we can compare with the "truth".

Spectrum of differences



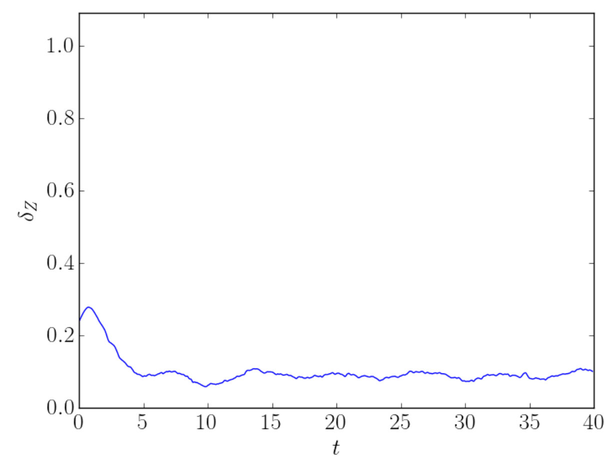
$$E_\Delta(k) = \sum_{|k|=k} |\hat{u} - \hat{u}_{\text{ref}}|^2(k)$$

Velocity correlations



$$\delta = \frac{\langle u \cdot u_{\text{ref}} \rangle}{\sqrt{\langle u^2 \rangle \langle u_{\text{ref}}^2 \rangle}}$$

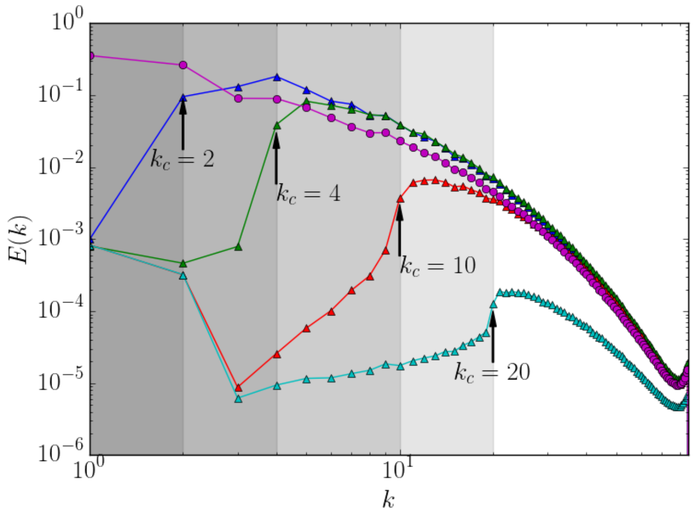
Vorticity correlations

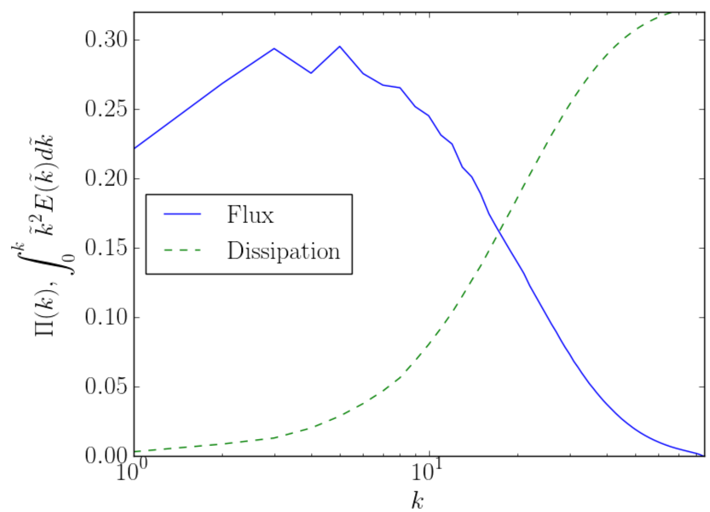


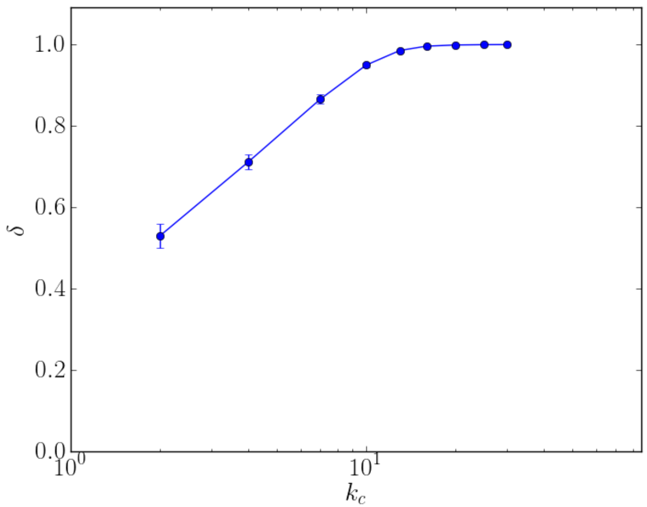
$$\delta_Z = \frac{\langle \omega \cdot \omega_{\text{ref}} \rangle}{\sqrt{\langle \omega_u^2 \rangle \langle \omega_{\text{ref}}^2 \rangle}}$$

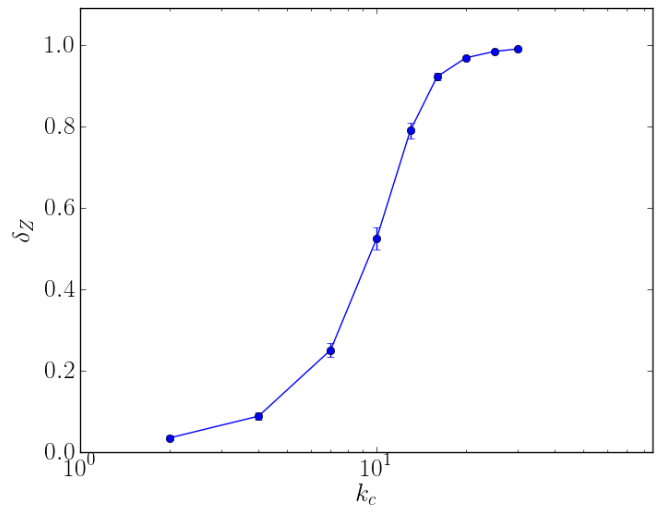
How much should one nudge a turbulent flow
in order to control (synchronize) it?

Nudging at different scales









In order to achieve good correlation in the large and the small scales one needs to nudge up to the scales when dissipation starts becoming important

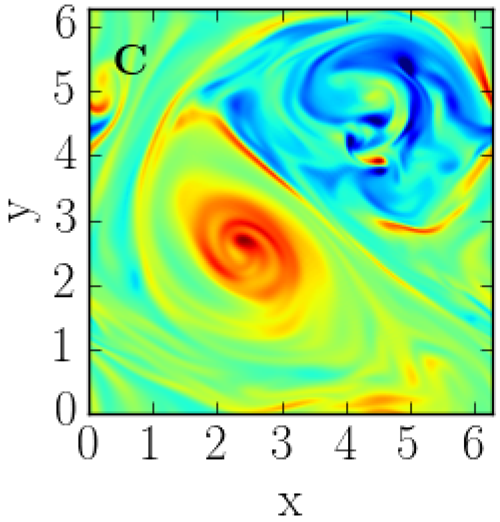
Smaller bound that the one coming from synchronization theory [Lalescu et al (2013)]

Note that the number of nudged modes is still small compared to the total

$$\left(\frac{17}{81}\right)^3 \approx 0.01$$

Spectral nudging as a physics based parameter estimator

$\Omega = ???$



Given data with unknown parameters, can we use nudging to impose correlations and find out these values?

We put this idea to test with a rotating turbulence flow:

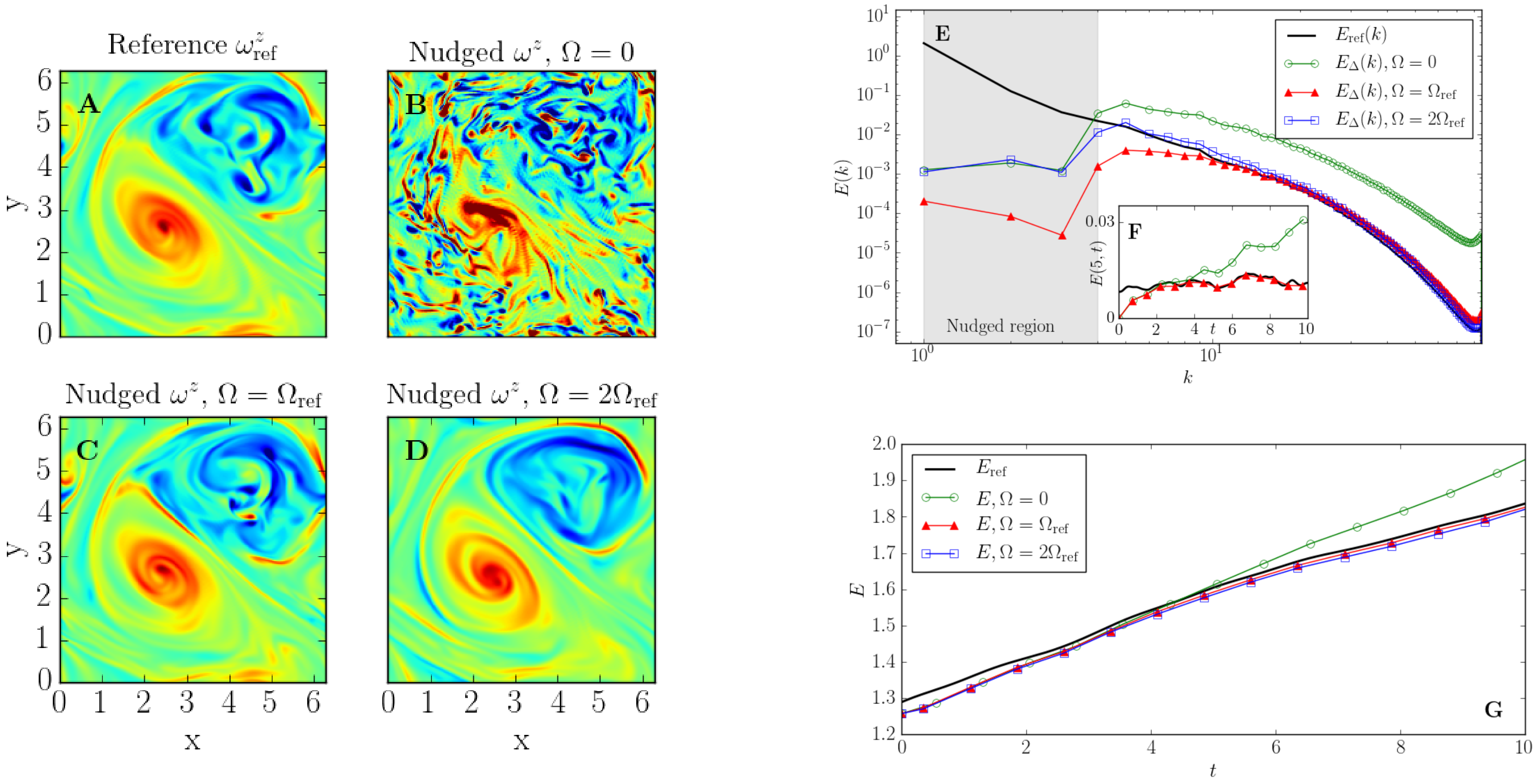
$$\frac{\partial u_{\rm ref}}{\partial t} + (u_{\rm ref} \cdot \nabla) u_{\rm ref} + 2\Omega_0 \times u_{\rm ref} = -\nabla p + \nu \nabla^2 u_{\rm ref} + f$$

We can run different nudged simulations varying the rotation frequency Ω

[Clark, Mazzino & Biferale, arXiv:1804.07680 (2018)]

Finding out parameters

Nudged simulations done varying the rotation frequency Ω
The reference has a frequency of Ω_0

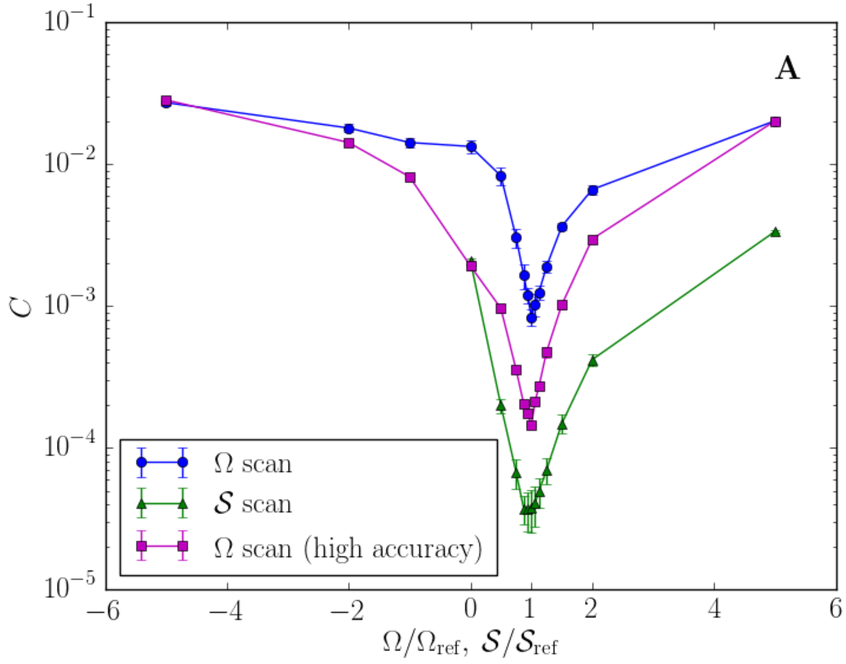


Nudging is sensitive to the choice of parameters! Very sensitive actually!

Quantifying the search

$$C = \int_0^{k_c} E_\Delta(k)dk$$

Integrate only over the scales at which we have information



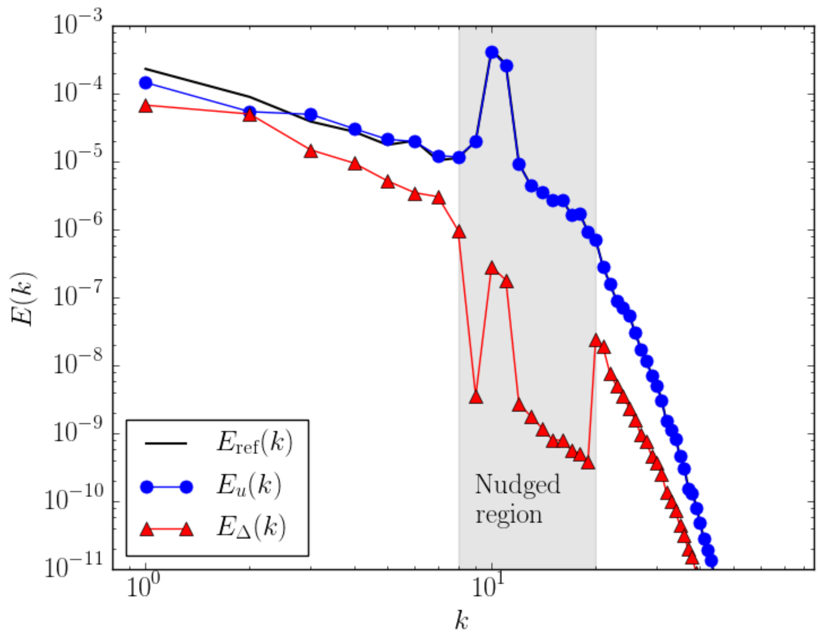
- Nudging can be used to determine the values of different parameters.
- Errors between 3% and 10%!
- It’s cheap, versatile and easy generalizable.
- No need to make assumptions such as linearity or quasi-Gaussianity.

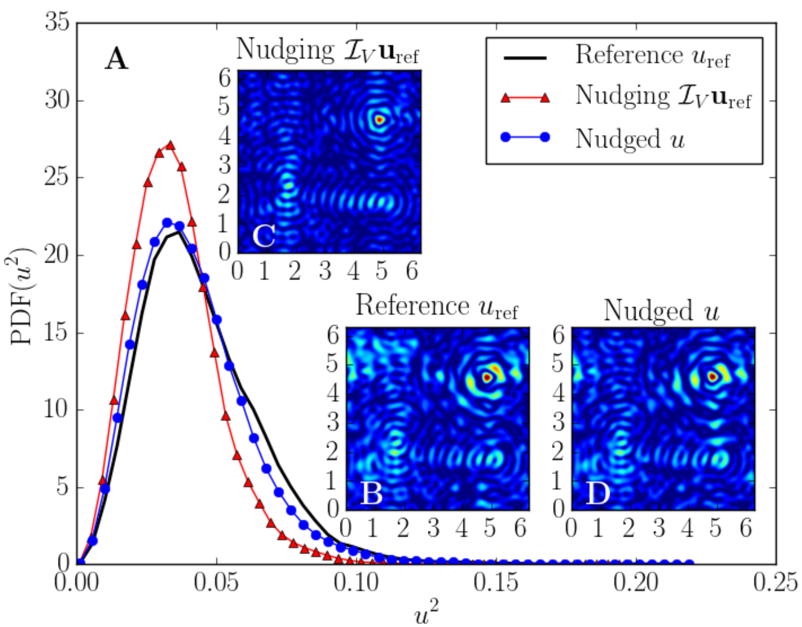
Nudging under an inverse cascade

So far we have seen that nudged systems can develop direct cascades.
But what happens when the reference simulation has a inverse cascade?

Can we infer physics as well as parameters?

We test this with a rotating turbulent flow force at k=10.

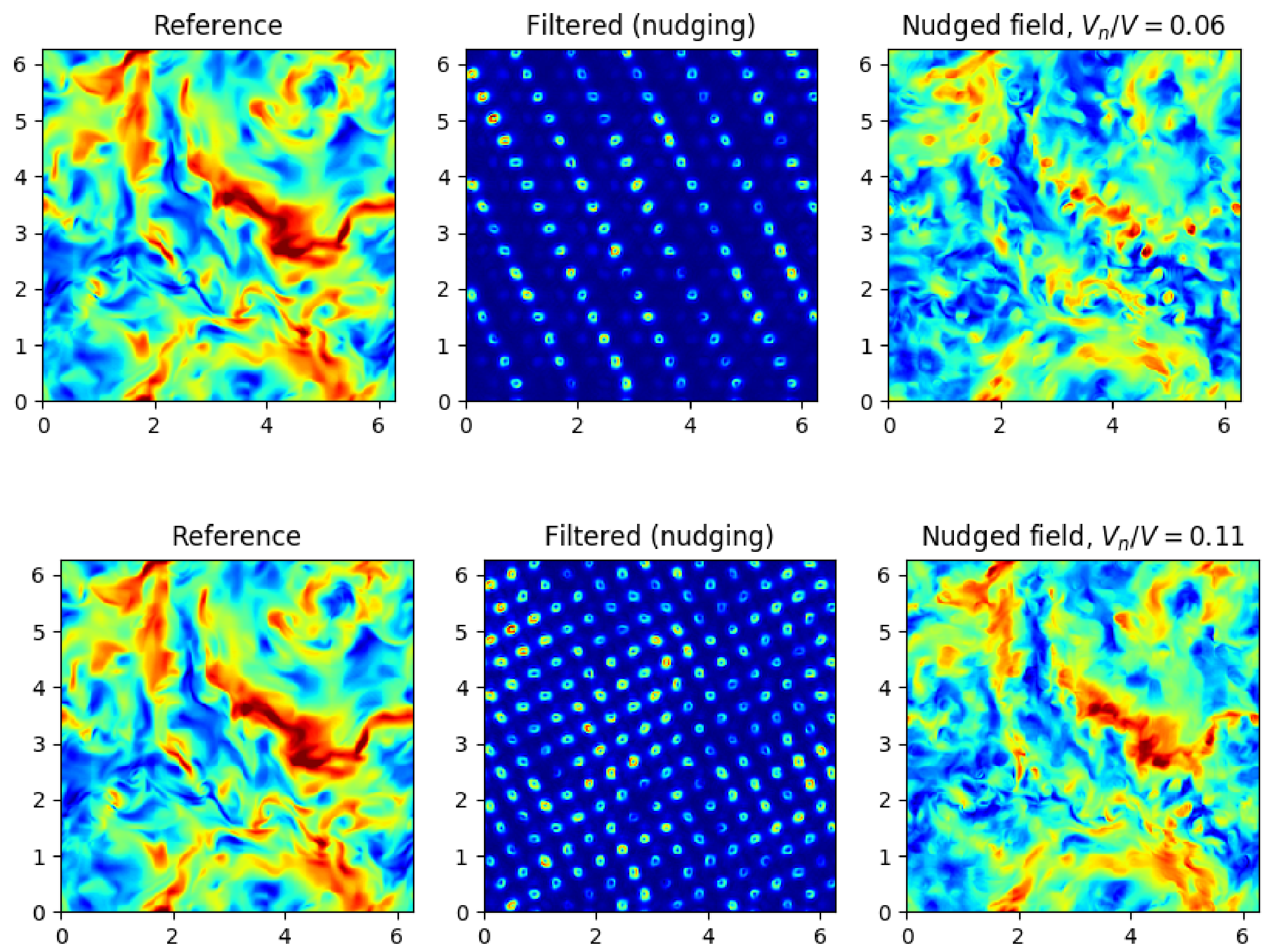




Nudging can then be used to learn about the *physics* of the reference flow

Even if the nudged simulation has no rotation, it can still recover the inverse cascade!

Upcoming: Nudging in real space



How large a volume do we have to nudge in order to synchronize?
Can we identify key regions in real space?

Conclusions

- We explored how the spectral nudging technique can be applied to fully developed three dimensional turbulence
- Multiscale turbulent flows can be completely synchronized. It requires information requires information on a lot of scales, but not on all!
- The nudging algorithm can be used to find out parameters from the reference data. The method is physics based and easily applicable to different flows.
- Nudging can also be used to learn about the physics of the reference flow.

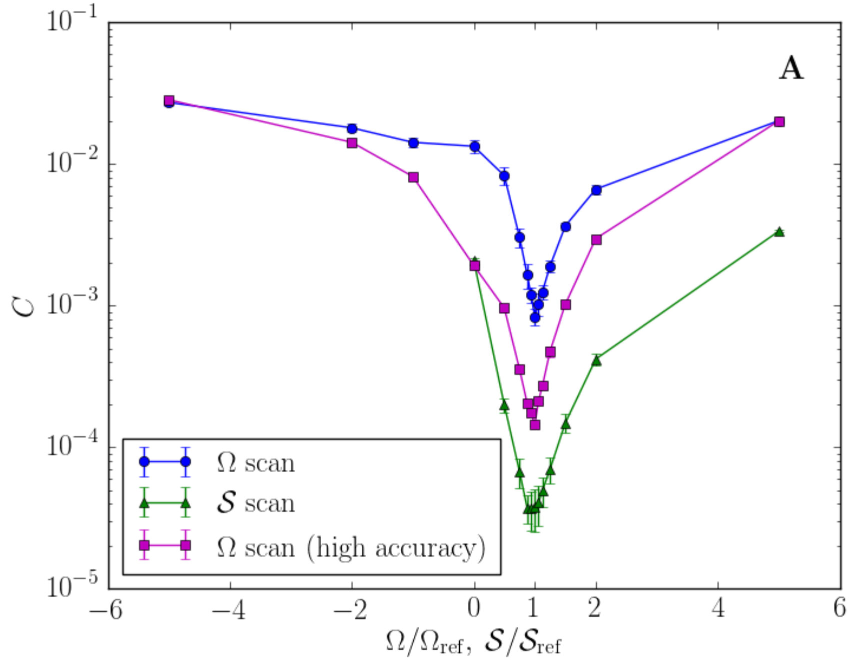
Nudging is a physics-informed way to probe the equations of motion and determine key degrees of freedom.

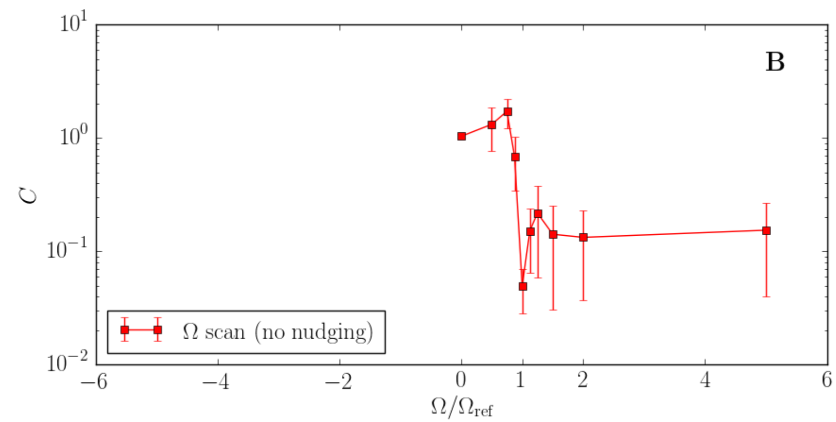
Thanks!

Quantifying the search

$$C = \int_0^{k_c} E_\Delta(k)dk$$

Integrate only over the scales at which we have information





Scan with no nudging (just same initial condition)

- Nudging can be used to determine the values of different parameters.
- Errors between 3% and 10%!
- It’s cheap, versatile and easy generalizable.
- No need to make assumptions such as linearity or quasi-Gaussianity.